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A Study on (*Q*,*L*)-fuzzy Normal ℓ-subsemiring of a ℓ-semiring Under Homomorphism and Anti-homomorphism

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Abstract: Our motivation in this paper is to start and study the algebraic nature of (Q,L)-fuzzy normal ℓ-subsemirings of a ℓ-semiring. Our purpose of this paper is to initiate and study the notions of (Q,L) - fuzzy normal ℓ-subsemirings of ℓ-semiring.We introduce the concept of (Q,L)-fuzzy normal ℓ-subsemirings of a ℓ-semiring. We study few of their elementary properties by (Q,L)-fuzzy normal ℓ-subsemirings of a ℓ-semiring and establish some results on these. These ideas are utilized in the improvement of some significant outcomes. We additionally made an endeavor to study the few properties of (Q,L)-fuzzy normal ℓ-subsemirings of ℓ-semiring under homomorphism and anti-homomorphism. Also some theorem is the composition operation of functions (Q,L)-fuzzy normal ℓ-subsemirings of ℓ-semiring under isomorphism and anti-isomorphism. Also prove some more properties of homomorphism and anti-homomorphism image and pre- image of (Q,L)-fuzzy normal ℓ-subsemirings of ℓ-semiring. Finally, we present the theorems of (Q,L)-fuzzy normal ℓ-subsemirings of ℓ-semiring under image and pre- image of isomorphism and anti- isomorphism.

Keywords: (Q,L)-fuzzy Subset, (Q,L)-fuzzy ℓ-subsemiring, (Q,L)-fuzzy Normal ℓ-subsemiring, Product of (Q,L)-fuzzy Subsets, Strongest (Q,L)-fuzzy Relation, Pseudo (Q,L)-fuzzy Coset

1. Introduction

The idea of Lattice was first characterized by Dedekind in 1897 and then doveloped by Birkhoft. G, imposed an operation an open problem "Is there a common abstraction which includes Boolean algebra, Boolean rings and lattice ordered group or L-bunch is a mathematical design associating lattice also, bunch. The notion of fuzzy sets was first introduced by L. A. Zadeh [18], several researchers explored on the generalization of the concept of fuzzy sets. The notion of fuzzy subnearrings and ideals was introduced by S. Abou Zaid [12]. Biswas. R [5], have introduced fuzzy subgroups and anti-fuzzy subgroups. Palaniappan, N. and Muthuraj, R. [9] have introduced the notion of homomorphism, anti-homomorphism of fuzzy and anti-fuzzy groups. Palaniappan. N & Arjunan. K, Operation on fuzzy and anti fuzzy ideals was introduced by Palaniappan, N. and Arjunan. K [8]. A new algebraic structure called Q-fuzzy subgroups was introduced by A. Solairaju and R. Nagarajan [4]. Saravanan. V and Sivakumar. D [14, 15] have introduced and defined a new algebraic structure of anti-fuzzy and anti-fuzzy normal subsemiring of a semiring. In this paper, we make some characterization of (Q,L)-fuzzy normal set and then proved some results on (Q,L)-fuzzy normal ℓ-subsemiring of a ℓ-semiring.

2. Preliminaries

Definition 2.1 Let X be a non-empty set and $L = (L, \leq)$ be a lattice with least element 0 and greatest element 1 and Q be a non-empty set. A (Q, L)-fuzzy subset A_μ of X is a function $A_\mu : X \times Q \rightarrow L$.

Definition 2.2 Let R be a ℓ-semiring and Q be a non empty set. A (Q, L)-fuzzy subset A of R is said to be a (Q, L)-fuzzy

ℓ-subsemiring ($QLFLSSR$) of R if the following conditions are satisfied:

(i) $A(x+y,q) \geq A(x,q) \wedge A(y,q)$,

(ii) $A(xy,q) \geq A(x,q) \wedge A(y,q)$,

(iii) $A(x \vee y,q) \geq A(x,q) \wedge A(y,q)$,

(iv) $A(x \wedge y,q) \geq A(x,q) \wedge A(y,q)$, for all x and y in R and q in Q.

Definition 2.3Let R be a ℓ-semiring and Q be a non-empty set. An (Q,L)-fuzzyℓ-subsemiring A of R is said to be an (Q,L)-fuzzy normal ℓ-subsemiring($QLFNLSSR$) of R if it satisfies the following conditions:

(i) $A(x+y,q) = A(y+x,q)$,

(ii) $A(xy,q) = A(yx,q)$,

(iii) $A(x \vee y,q) = A(y \vee x,q)$,

(iv) $A(x \wedge y,q) = A(y \wedge x,q)$, for all x and y in R and q in Q.

Definition 2.4 Let A and B be any two (Q,L)-fuzzy subsets of sets G and H, respectively. The product of A and B, denoted by $A \times B$, is defined as $A \times B = \{< ((x,y),q), A \times B\ ((x,y),q) >/$ for all x in R and y in H and q in $Q\}$, where $A \times B((x,y),q) = A(x,q) \wedge B(y,q)$.

Definition 2.5Let R and R' be any two ℓ-semirings Q be a non empty set. Let $f: R \to R'$ be any function and A be a (Q,L)-fuzzy ℓ-subsemiring in R, V be a (Q,L)-fuzzy ℓ-subsemiring in $f(R) = R'$, defined by $V(y,q) = \sup_{x \in f^{-1}(y)} A(x,y)$, for all x in R and y in $R'$and q in Q.Then A is called a pre-image of V under f and is denoted by $f^{-1}(V)$.

Definition 2.6 Let A be a (Q,L)-fuzzy subset in a set S, the strongest (Q,L)-fuzzy relation on S, that is a (Q,L)-fuzzy relation V with respect to A given by $V((x,y),q) = A(x,q) \wedge A(y,q)$, for all x and y in S and q in Q.

Definition 2.7 A (Q,L)-fuzzy subset A of a set X is said to be normalized if there exists an element x in X such that $A(x,q) = 1$.

Definition 2.8 Let A be an (Q,L)-fuzzy ℓ-subsemiring of a ℓ-semiring R and a in R. Then the pseudo (Q,L)-fuzzy coset $(aA)^p$ is defined by $((aA)^p)(x,q) = p(a)A(x,q)$, for every x in R and for some p in P and q in Q.

Definition 2.9 Let A be a (Q,L)-fuzzy subset of X. Forα in L, a Q-level subset of A is the set $A_\alpha = \{ x \in X: A(x,q) \geq \alpha\}$.

3. Properties of (Q,L)-fuzzy Normal ℓ-subsemiringofa ℓ-semiring

Theorem 3.1 Let R be a ℓ-semiring Q be a non-empty set. If A and B are two(Q,L)-fuzzy normal ℓ-subsemirings of R, then their intersection $A \cap B$ is an (Q,L)-fuzzy normal ℓ-subsemiring of R.

Proof: Let x and $y \in R$.Let $A = \{\langle (x,q), A(x,q)\rangle / x$ in R and q in $Q\}$ and $B = \{\langle (x,q), B(x,q)\rangle / x$ in R and q in $Q\}$ be (Q,L)-fuzzy normalℓ-subsemirings of a ℓ-semiring R.Let $C = A \cap B$ and $C = \{\langle (x,q), C(x,q)\rangle / x$ in R and q in $Q\}$.Then, Clearly C is an (Q,L)-fuzzy ℓ-subsemiring of a ℓ-semiring R, since A and B are two (Q,L)-fuzzy ℓ-subsemirings of a ℓ-semiring R and,(i) $C(x+y,q) = A(x+y,q) \wedge B(x+y,q) = A(y+x,q) \wedge B(y+x,q) = C(y+x,q)$, for all$x$ and y in R and q in Q.Therefore,$C(x+y,q) = (y+x,q)$, for all x and y in R and q in Q. (ii) $C(xy,q) = A(xy,q) \wedge B(xy,q) = A(yx,q) \wedge B(yx,q) = C(yx,q)$, for all x and y in R and q in Q.Therefore, $C(xy,q) = C(yx,q)$, for all x and y in R and q in Q. Also, (iii) $C(x \vee y,q) = A(x \vee y,q) \wedge B(x \vee y,q) = A(y \vee x,q) \wedge B(y \vee x,q) = C(y \vee x,q)$, for all x and y in $R$and q in Q. Therefore, $C(x \vee y,q) = (y \vee x,q)$, for all x and y in R and q in Q. (iv) $C(x \wedge y,q) = A(x \wedge y,q) \wedge B(x \wedge y,q) = A(y \wedge x,q) \wedge B(y \wedge x,q) = C(y \wedge x,q)$, for all x and y in R and q in Q. Therefore,$C(x \wedge y,q) = (y \wedge x,q)$, for all x and y in R and q in Q.Hence $A \cap B$ is an (Q,L)-fuzzy normal ℓ-subsemiring of aℓ-semiring$R$.

Theorem 3.2Let R be a ℓ-semiring Q be a non-empty set. The intersection of a family of (Q,L)-fuzzy normal ℓ-subsemirings of R is an (Q,L)-fuzzy normalℓ-subsemiring of R.

Proof: Let$\{A_i\}_{i\in I}$be a family of(Q,L)-fuzzy normal ℓ-subsemirings of a ℓ-semiring R andlet

$$A = \bigcap_{i \in I} A_i.$$

Then for x and y in R and q in Q.Clearly the intersection of a family of (Q,L)-fuzzy ℓ-subsemirings of a ℓ-semiring R is an (Q,L)-fuzzy ℓ-subsemiring of a ℓ-semiring R. (i) $A(x+y,q) = \inf_{x \in f^{-1}(y)} A_i(x+y,q) = \inf_{x \in f^{-1}(y)} A_i(y+x,q) = A(y+x,q)$. Therefore, $A(x+y,q) = A(y+x,q)$, for all x and y in R and q in Q. (ii) $A(xy,q) = \inf_{x \in f^{-1}(y)} A_i(xy,q) = \inf_{x \in f^{-1}(y)} A_i(yx,q) = A(yx,q)$. Therefore, $A(xy,q) = A(yx,q)$, for all x and y in R and q in Q. (iii) $A(x \vee y,q) = \inf_{x \in f^{-1}(y)} A_i(x \vee y,q) = \inf_{x \in f^{-1}(y)} A_i(y \vee x,q) = A(y \vee x,q)$. Therefore, $A(x \vee y,q) = A(y \vee x,q)$, for all x and y in R and q in Q. (iv) $A(x \wedge y,q) = \inf_{x \in f^{-1}(y)} A_i(x \wedge y,q) = \inf_{x \in f^{-1}(y)} A_i(y \wedge x,q) = A(y \wedge x,q)$. Therefore, $A(x \wedge y,q) = A(y \wedge x,q)$, for all x and y in R and q in Q.Hence the intersectionof a family of (Q,L)-fuzzy normal ℓ-subsemirings of a ℓ-semiring$R$ is an (Q,L)-fuzzy normalℓ-subsemiring of a ℓ-semiring$R$.

Theorem3.3 Let A and B be (Q,L)-fuzzy ℓ-subsemiring of the ℓ-semirings G and H, respectively. If A and B are (Q,L)-fuzzy normal ℓ-subsemirings.Then$A \times B$ is an(Q,L)- fuzzy normal ℓ-subsemiring of $G \times H$.

Proof: Let A and B be (Q,L)-fuzzy normal ℓ-subsemirings of the ℓ-semirings G and H respectively.Clearly $A \times B$ is an (Q,L)-fuzzy ℓ-subsemiring of $G \times H$. Let x_1 and x_2 be in G, y_1 and y_2 be in H, q in Q.Then (x_1,y_1) and (x_2,y_2) are in $G \times H$. Now, $A \times B[(x_1,y_1)+(x_2,y_2),q] = A \times B\big((x_1+x_2, y_1+y_2),q\big) = A(x_1+x_2,q) \wedge B(y_1+y_2,q) = A(x_2+x_1,q) \wedge B(y_2+y_1,q) = A \times B\big((x_2+x_1, y_2+$

$y_1), q) = A \times B[(x_2, y_2) + (x_1, y_1), q]$. Therefore, $A \times B[(x_1, y_1) + (x_2, y_2), q] = A \times B[(x_2, y_2) + (x_1, y_1), q]$. And, $A \times B\,[(x_1, y_1)(x_2, y_2), q] = A \times B((x_1x_2, y_1y_2), q) = A(x_1x_2, q) \wedge B(y_1y_2, q) = A(x_2x_1, q), B(y_2y_1, q) = A \times B[(x_2, y_2)(x_1, y_1), q]$. Therefore, $A \times B[(x_1, y_1)(x_2, y_2), q] = A \times B[(x_2, y_2)(x_1, y_1), q]$. Also, $A \times B\,[(x_1, y_1) \vee (x_2, y_2), q] = A \times B((x_1 \vee x_2, y_1 \vee y_2), q) = A(x_1 \vee x_2, q) \vee B(y_1 \vee y_2, q) = A(x_2 \vee x_1, q) \wedge B(y_2 \vee y_1, q) = A \times B((x_2 \vee x_1, y_2 \vee y_1), q) = A \times B[(x_2, y_2) \vee (x_1, y_1), q]$. Therefore, $A \times B[(x_1, y_1) \vee (x_2, y_2), q] = A \times B[(x_2, y_2) \vee (x_1, y_1), q]$. And, $A \times B\,[(x_1, y_1) \wedge (x_2, y_2), q] = A \times B((x_1 \wedge x_2, y_1 \wedge y_2), q) = A(x_1 \wedge x_2, q) \quad B(y_1 \wedge y_2, q) = A(x_2 \wedge x_1, q) \wedge B(y_2 \wedge y_1, q) = A \times B((x_2 \wedge x_1, y_2 \wedge y_1), q) = A \times B[(x_2, y_2) \wedge (x_1, y_1), q]$. Therefore, $A \times B[(x_1, y_1) \wedge (x_2, y_2), q] = A \times B[(x_2, y_2) \wedge (x_1, y_1), q]$. Hence $A \times B$ is an (Q, L)-fuzzy normal ℓ-subsemiring of $G \times H$.

Theorem 3.4 Let A be a fuzzy subset in a ℓ-semiring R and V be the strongest (Q, L)-fuzzy relation on R. Then A is an (Q, L)-fuzzy normal ℓ-subsemiring of R if and only if V is an (Q, L)-fuzzy normal ℓ-subsemiring of $R \times R$.

Proof: Suppose that A is a (Q, L)-fuzzy normal ℓ-subsemiring of R. Then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in $R \times R$ and q in Q. Clearly V is a (Q, L)-fuzzy ℓ-subsemiring of $R \times R$. We have, $V(x + y, q) = V[(x_1, x_2) + (y_1, y_2), q] = V((x_1 + y_1, x_2 + y_2), q) = A((x_1 + y_1), q) \wedge A((x_2 + y_2), q) = A((y_1 + x_1), q) \wedge A((y_2 + x_2), q) = V((y_1 + x_1, y_2 + x_2), q) = V[(y_1, y_2) + (x_1, x_2), q] = V(y + x, q)$. Therefore, $V(x + y, q) = V(y + x, q)$, for all x and y in $R \times R$ and q in Q. We have, $V(xy, q) = V[(x_1, x_2)(y_1, y_2), q] = V((x_1y_1, x_2y_2), q) = A((x_1y_1), q) \wedge A((x_2y_2), q) = A((y_1x_1), q) \wedge A((y_2x_2), q) = V((y_1x_1, y_2x_2), q) = V[(y_1, y_2)(x_1, x_2), q] = V(yx, q)$ Therefore, $V(xy, q) = V(yx, q)$, for all x and y in $R \times R$ and q in Q. Also, $V(x \vee y, q) = V[(x_1, x_2) \vee (y_1, y_2), q] = V((x_1 \vee y_1, x_2 \vee y_2), q) = A((x_1 \vee y_1), q) \wedge A((x_2 \vee y_2), q) = A((y_1 \vee x_1), q) \wedge A((y_2 \vee x_2), q) = V((y_1 \vee x_1, y_2 \vee x_2), q) = V[(y_1, y_2) \vee (x_1, x_2), q] = V(y \vee x, q)$. Therefore, $V(x \vee y, q) = V(y \vee x, q)$, for all x and y in $R \times R$ and q in Q. And, $V(x \wedge y, q) = V[(x_1, x_2) \wedge (y_1, y_2), q] = V((x_1 \wedge y_1, x_2 \wedge y_2), q) = A((x_1 \wedge y_1), q) \wedge A((x_2 \wedge y_2), q) = A((y_1 \wedge x_1), q) \wedge A((y_2 \wedge x_2), q) = V((y_1 \wedge x_1, y_2 \wedge x_2), q) = V[(y_1, y_2) \wedge (x_1, x_2), q] = V(y \wedge x, q)$ Therefore, $V(x \wedge y, q) = V(y \wedge x, q)$, for all x and y in $R \times R$ and q in Q. This proves that V is a (Q, L)-fuzzy normal ℓ-subsemiring of $R \times R$. Conversely, assume that V is a (Q, L)-fuzzy normal ℓ-subsemiring of $R \times R$, then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in $R \times R$, we have $A(x_1 + y_1, q) \wedge A(x_2 + y_2, q) = V((x_1 + y_1, x_2 + y_2), q) = V[(x_1, x_2) + (y_1, y_2), q] = V(x + y, q) = V(y + x, q) = V[(y_1, y_2) + (x_1, x_2), q] = V((y_1 + x_1, y_2 + x_2), q) = A(y_1 + x_1, q) \wedge A(y_2 + x_2, q)$. We get, $A((x_1 + y_1), q) = A((y_1 + x_1), q)$, for all x_1 and y_1 in R and q in Q. And $A(x_1y_1, q) \wedge A(x_2y_2, q) = V((x_1y_1, x_2y_2), q) = V[(x_1, x_2)(y_1, y_2), q] = V(xy, q) = V(yx, q) = V[(y_1, y_2)(x_1, x_2), q] = V((y_1x_1, y_2x_2), q) = A(y_1x_1, q) \wedge A(y_2x_2, q)$. We get, $A((x_1y_1), q) = A((y_1x_1), q)$, for all x_1 and y_1 in R and q in Q. Also, $A(x_1 \vee y_1, q) \wedge A(x_2 \vee y_2, q) = V((x_1 \vee y_1, x_2 \vee y_2), q) = V[(x_1, x_2) \vee (y_1, y_2), q] = V(x \vee y, q) = V(y \vee x, q) = V[(y_1, y_2) \vee (x_1, x_2), q] = V((y_1 \vee x_1, y_2 \vee x_2), q) = A(y_1 \vee x_1, q) \wedge A(y_2 \vee x_2, q)$. We get, $A((x_1 \vee y_1), q) = A((y_1 \vee x_1), q)$, for all x_1 and y_1 in R and q in Q. And, $A(x_1 \wedge y_1, q) \wedge A(x_2 \wedge y_2, q) = V((x_1 \wedge y_1, x_2 \wedge y_2), q) = V[(x_1, x_2) \wedge (y_1, y_2), q] = V(x \wedge y, q) = V(y \wedge x, q) = V[(y_1, y_2) \wedge (x_1, x_2), q] = V((y_1 \wedge x_1, y_2 \wedge x_2), q) = A(y_1 \wedge x_1, q) \wedge A(y_2 \wedge x_2, q)$. We get, $A((x_1 \wedge y_1), q) = A((y_1 \wedge x_1), q)$, for all x_1 and y_1 in R and q in Q. Hence A is a (Q, L)-fuzzy normal ℓ-subsemiring of R.

Theorem 3.5 Let R and R' be any two ℓ-semirings and Q be a non-empty set. The homomorphic image of an (Q, L)-fuzzy normal ℓ-subsemiring of R is an (Q, L)-fuzzy normal ℓ-subsemiring of R'.

Proof: Let R and R' be any two ℓ-semirings Q be a non-empty set and $f: R \to R'$ be a homomorphism. Then, $f(x + y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$, for all x and y in R. Let $V = f(A)$, where A is an (Q, L)-fuzzy normal ℓ-subsemiring of a ℓ-semiring R. We have to prove that V is an (Q, L)-fuzzy normal ℓ-subsemiring of a ℓ-semiring R'. Now, for $f(x), f(y)$ in R', clearly V is an (Q, L)-fuzzy ℓ-subsemiring of a ℓ-semiring R', since A is an (Q, L)-fuzzy ℓ-subsemiring of a ℓ-semiring R. Now, $V(f(x) + f(y), q) = V(f(x + y), q) \geq A(x + y, q) = A(y + x, q) \leq V(f(y + x), q) = V(f(y) + f(x), q)$. Therefore, $V(f(x) + f(y), q) = V(f(y) + f(x), q)$, for all $f(x)$ and $f(y)$ in R'. Again, $V(f(x)f(y), q) = V(f(xy), q) \geq A(xy, q) = A(yx, q) \leq V(f(yx), q) = V(f(y)f(x), q)$. Therefore, $V(f(x)f(y), q) = V(f(y)f(x), q)$, for all $f(x)$ and $f(y)$ in R'. Also, $V(f(x) \vee f(y), q) = V(f(x \vee y), q) \geq A(x \vee y, q) = A(y \vee x, q) \leq V(f(y \vee x), q) = V(f(y) \vee f(x), q)$. Therefore, $V(f(x) \vee f(y), q) = V(f(y) \vee (f(x), q)$, for all $f(x)$ and $f(y)$ in R'. And, $V(f(x) \wedge f(y), q) = V(f(x \wedge y), q) \geq A(x \wedge y, q) = A(y \wedge x, q) \leq V(f(y \wedge x), q) = V(f(y) \wedge f(x), q)$. Therefore, $V(f(x) \wedge f(y), q) = V(f(y) \wedge (f(x), q)$, for all $f(x)$ and $f(y)$ in R'. Hence V is an (Q, L)-fuzzy normal ℓ-subsemiring of a ℓ-semiring R'.

Theorem 3.6Let R and R' be any two ℓ-semirings and Q be a non-empty set. The homomorphicpreimage of an (Q, L)-fuzzy normal ℓ-subsemiring of R' is an (Q, L)-fuzzy normal ℓ-subsemiring of R.

Proof: Let R and R' be any two ℓ-semirings Q be a non-empty set and $f: R \to R'$ be a homomorphism. Then, $f(x + y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$, for all x and y in R. Let $V = f(A)$, where V is an (Q, L)-fuzzy normal ℓ-subsemiring of a ℓ-semiring R'. We have to prove that A is an (Q, L)-fuzzy normal ℓ-subsemirring of a ℓ-semiring R. Let x and y in R. Then, clearly A is an (Q, L)-fuzzy ℓ-subsemiring of a ℓ-semiring R, since V is an (Q, L)-fuzzy ℓ-subsemiring of a ℓ-semiringR'. Now, $A(x + y, q) = V(f(x + y), q) = V(f(x) + f(y), q) = V(f(y) + f(x), q) = V(f(y + x), q) = A(y + x, q)$. Therefore, $A(x + y, q) = A(y + x, q)$, for all x and y in R and q in Q. Again, $A(xy, q) = V(f(xy), q) = V(f(x)f(y), q) = V(f(y)f(x), q) = V(f(yx), q) = A(yx, q)$. Therefore, $A(xy, q) = A(yx, q)$, for all x and y in R and q in

Q. Also, $A(x \vee y, q) = V(f(x \vee y), q) = V(f(x) \vee f(y), q) = V(f(y) \vee f(x), q) = V(f(y \vee x), q) = A(y \vee x, q)$. Therefore, $A(x \vee y, q) = A(y \vee x, q)$, for all x and y in R and q in Q. And, $A(x \wedge y, q) = V(f(x \wedge y), q) = V(f(x) \wedge f(y), q) = V(f(y) \wedge f(x), q) = V(f(y \wedge x), q) = A(y \wedge x, q)$. Therefore, $A(x \wedge y, q) = A(y \wedge x, q)$, for all x and y in R and q in Q.Hence A is an (Q, L)-fuzzy normal ℓ-subsemiring of a ℓ-semiring R.

Theorem 3.7Let R and R' be any two ℓ-semirings and Q be a non-empty set. The anti-homomorphic image of an (Q, L)-fuzzy normal ℓ-subsemiring of R is an (Q, L)-fuzzy normal ℓ-subsemiring of R'.

Proof: Let R and R' be any two ℓ-semirings and Q be a non-empty setand $f: R \to R'$ be an anti-homomorphism. Then, $f(x + y) = f(y) + f(x)$ and $f(xy) = f(y)f(x)$, for all x and y in R.Let $V = f(A)$, where A is an (Q, L)-fuzzy normal ℓ-subsemiring of a ℓ-semiring$R$. We have to prove that V is an (Q, L)-fuzzy normal ℓ-subsemiring of a ℓ-semiringR'.Now, for $f(x)$ and $f(y)$ in R', clearly V is an (Q, L)-fuzzy ℓ-subsemiring of a ℓ-semiring R', since A is an (Q, L)-fuzzy ℓ-subsemiring of a ℓ-semiring R.Now,$V(f(x) + f(y), q) = V(f(y + x), q) \geq A(y + x, q) = A(x + y, q) \leq V(f(x + y), q) = V(f(y) + f(x), q)$. Therefore, $V(f(x) + f(y), q) = V(f(y) + f(x), q)$, for all $f(x)$ and $f(y)$ in R'. Again, $V(f(x)f(y), q) = V(f(yx), q) \geq A(yx, q) = A(xy, q) \leq V(f(xy), q) = V(f(y)f(x), q)$. Therefore, $(f(x)f(y), q) = V(f(y)f(x), q)$, for all $f(x)$ and $f(y)$ in R'. Also, $V(f(x) \vee f(y), q) = V(f(y \vee x), q) \geq A(y \vee x, q) = A(x \vee y, q) \leq V(f(x \vee y), q) = V(f(y) \vee f(x), q)$. Therefore, $V(f(x) \vee f(y), q) = V(f(y) \vee f(x), q)$, for all $f(x)$ and $f(y)$ in R'. And, $V(f(x) \wedge f(y), q) = V(f(y \wedge x), q) \geq A(y \wedge x, q) = A(x \wedge y, q) \leq V(f(x \wedge y), q) = V(f(y) \wedge f(x), q)$. Therefore, $V(f(x) \wedge f(y), q) = V(f(y) \wedge f(x), q)$, for all $f(x)$ and $f(y)$ in R'. Hence V is an (Q, L)-fuzzy normal ℓ-subsemiring of a ℓ-semiring R'.

Theorem 3.8Let R and R' be any two ℓ-semirings and Q be a non-empty set. The anti-homomorphicpreimage of an (Q, L)-fuzzy normal ℓ-subsemiring of R' is an (Q, L)-fuzzy normal ℓ-subsemiring of R.

Proof: Let$R$ and R' be any twoℓ-semirings and Q be a non-empty setand $f: R \to R'$ be an anti-homomorphism. Then, $f(x + y) = f(y) + f(x)$ and $f(xy) = f(y)f(x)$, for all x and y in R.Let $V = f(A)$, where V is an (Q, L)-fuzzy normal ℓ-subsemiring of a ℓ-semiring R'.We have to prove that A is an (Q, L)-fuzzy normal ℓ-subsemiring of a ℓ-semiring R.Let x and y in R, then clearly A is an (Q, L)-fuzzy ℓ-subsemiring of a ℓ-semiring R, since V is an (Q, L)-fuzzyℓ-subsemiring of a ℓ -semiring R'. Now, $A(x + y, q) = V(f(x + y), q) = V(f(y) + f(x), q) = V(f(x) + f(y), q) = V(f(y + x), q) = A(y + x, q)$. Therefore, $A(x + y, q) = A(y + x, q)$, for all x and y in R and q in Q. Again, $A(xy, q) = V(f(xy), q) = V(f(y)f(x), q) = V(f(x)f(y), q) = V(f(yx), q) = A(yx, q)$. Therefore, $A(xy, q) = A(yx, q)$, for all x and y in R and q in Q. Also, $A(x \vee y, q) = V(f(x \vee y), q) = V(f(y) \vee f(x), q) = V(f(x) \vee f(y), q) = V(f(y \vee x), q) = A(y \vee x, q)$.Therefore,$A(x \vee y, q) = A(y \vee x, q)$, for all x and y in R and q in Q. And, $A(x \wedge y, q) = V(f(x \wedge y), q) = V(f(y) \wedge f(x), q) = V(f(x) \wedge f(y), q) = V(f(y \wedge x), q) = A(y \wedge x, q)$. Therefore, $A(x \wedge y, q) = A(y \wedge x, q)$, for all x and y in R and q in Q.Hence A is an (Q, L)-fuzzy normal ℓ-subsemiring of a ℓ-semiring R.

Theorem 3.9 Let A be an (Q, L)-fuzzy normal ℓ-subsemiring of a ℓ-semiring R, then the pseudo (Q, L)-fuzzy coset$(aA)^p$ is an (Q, L)-fuzzy normalℓ-subsemiringof a ℓ-semiring$R$, for a in $R$and q in Q.

Proof: Let A be an (Q, L)-fuzzy normal ℓ-subsemiring of a ℓ-semiring R. For every x and y in $R$and q in Q, we have, $((aA)^p)(x + y) = p(a)A(x + y) \geq p(a)\{(A(x) \wedge A(y)\} = \{p(a)A(x) \wedge p(a)A(y)\} = \{((aA)^p)(x) \wedge ((aA)^p)(y)\}$. Therefore, $((aA)^p)(x + y) = \{((aA)^p)(x) \wedge ((aA)^p)(y)\}$. Now, $((aA)^p)(xy) = p(a)A(xy) \geq p(a)\{A(x) \wedge A(y)\} = \{p(a)A(x) \wedge p(a)A(y)\} = \{((aA)^p)(x) \wedge ((aA)^p)(y)\}$. Therefore, $((aA)^p)(xy) = \{((aA)^p)(x) \wedge ((aA)^p)(y)\}$. Also, $((aA)^p)(x \vee y) = p(a)A(x \vee y) \geq p(a)\{(A(x) \wedge A(y)\} = \{p(a)A(x) \wedge p(a)A(y)\} = \{((aA)^p)(x) \wedge ((aA)^p)(y\}$. Therefore, $((aA)^p)(x \vee y) = \{((aA)^p)(x) \wedge ((aA)^p)(y)\}$. And, $((aA)^p)(x \wedge y) = p(a)A(x \wedge y) \geq p(a)\{(A(x) \wedge A(y)\} = \{p(a)A(x) \wedge p(a)A(y)\} = \{((aA)^p)(x) \wedge ((aA)^p)(y)\}$. Therefore, $((aA)^p)(x \wedge y) = \{((aA)^p)(x) \wedge ((aA)^p)(y)\}$. Hence $(aA)^p$ is an (Q, L)-fuzzy normal ℓ-subsemiring of a ℓ-semiring R.

Theorem 3.10 Let A and B be (Q, L)-fuzzy subsets of the sets R and H respectively, and let α in L. Then $(A \times B)_\alpha = A_\alpha \times B_\alpha$.

Proof: Let α in L. Let (x, y) be in $(\mathrm{A} \times \mathrm{B})_\alpha$ if and only if $A \times B((x, y), q) \geq \alpha$, if and only if $\{A(x, q) \wedge B(x, q)\} \geq \alpha$, if and only if $A(x, q) \geq \alpha$ and $B(x, q) \geq \alpha$, if and only if $x \in A_\alpha$ and $y \in B_\alpha$, if and only if $(x, y) \in A_\alpha \times B_\alpha$.Therefore,$(\mathrm{A} \times \mathrm{B})_\alpha = A_\alpha \times B_\alpha$.

Theorem 3.11 Let A be a (Q, L)-fuzzy normal ℓ-subsemiring of a ℓ-semiring R. If $A(x, q) < A(y, q)$, for some x and y in R and q in Q, then $A(x + y, q) = A(x, q) = A(y + x, q)$, for some x and y in R and q in Q.

Proof: It is trivial.

Theorem 3.12 Let A be a (Q, L)-fuzzy normal ℓ-subsemiring of a ℓ-semiring R. If $A(x, q) > A(y, q)$, for some x and y in R and q in Q, then $A(x + y, q) = A(y, q) = A(y + x, q)$, for some x and y in R and q in Q.

Proof: It is trivial.

4. In the Following Theorem Is the Composition Operation of Functions

Theorem 4.1 Let A be an (Q, L)-fuzzy normal ℓ-subsemiring of a ℓ-semiring H and f is an Isomorphism from a ℓ-semiring R onto H. Then $A \circ f$ is an (Q, L)-fuzzy Normal ℓ-subsemiring of the ℓ-semiring R.

Proof: Let$x$ and y in R and A be an (Q, L)-fuzzy normal ℓ-subsemiring of aℓ-semiring H.Then clearly $\mathrm{A} \circ \mathrm{f}$ is an (Q, L)-fuzzy ℓ-subsemiring of a ℓ-semiring R. Now, $(\mathrm{A} \circ \mathrm{f})(x + y, q) = A(f(x + y), q) = A(f(x) + f(y), q) = A(f(y) + f(x), q) = A(f(y + x), q) = (\mathrm{A} \circ \mathrm{f})(y + x, q)$. Therefore, $(\mathrm{A} \circ \mathrm{f})(x + y, q) = (\mathrm{A} \circ \mathrm{f})(y + x, q)$, for all x and y in R and q in Q. And,

$(A \circ f)(xy,q) = A(f(xy),q) = A(f(x)f(y),q) = A(f(y)f(x),q) = A(f(yx),q) = (A \circ f)(yx,q)$. Therefore $(A \circ f)(xy,q) = (A \circ f)(yx,q)$, for all x and y in R and q in Q.Also, $(A \circ f)(x \vee y,q) = A(f(x \vee y),q) = A(f(x) \vee f(y),q) = A(f(y) \vee f(x),q) = A(f(y \vee x),q) = (A \circ f)(y \vee x,q)$. Therefore, $(A \circ f)(x \vee y,q) = (A \circ f)(y \vee x,q)$, for all x and y in R and q in Q. And, $(A \circ f)(x \wedge y,q) = A(f(x \wedge y),q) = A(f(x) \wedge f(y),q) = A(f(y) \wedge f(x),q) = A(f(y \wedge x),q) = (A \circ f)(y \wedge x,q)$. Therefore, $(A \circ f)(x \wedge y,q) = (A \circ f)(y \wedge x,q)$, for all x and y in R and q in Q. Hence $A \circ f$ is an (Q,L)-fuzzy normal ℓ-subsemiring of a ℓ-semiring R.

Theorem 4.2 Let A be an (Q,L) -fuzzyNormal ℓ -subsemiring of a ℓ-semiring H and f is an anti-isomorphism from a ℓ-semiring R onto H. Then $A \circ f$ is an (Q,L)-fuzzy Normal ℓ-subsemiring of the ℓ-semiring R.

Proof: Let$x$ and y in R and A be an (Q,L)-fuzzy normal ℓ-subsemiring of a ℓ-semiring H. Then clearly $A \circ f$ is an (Q,L) -fuzzy ℓ-subsemiring of a ℓ-semiring R. Now, $(A \circ f)(x+y,q) = A(f(x+y),q) = A(f(y)+f(x),q) = A(f(x)+f(y),q) = A(f(y+x),q) = (A \circ f)(y+x,q)$. Therefore, $(A \circ f)(x+y,q) = (A \circ f)(y+x,q)$, for all x and y in R and q in Q. And, $(A \circ f)(xy,q) = A(f(xy),q) = A(f(y)f(x),q) = A(f(x)f(y),q) = A(f(yx),q) = (A \circ f)(yx,q)$. Therefore, $(A \circ f)(xy,q) = (A \circ f)(yx,q)$, for all x and y in R and q in Q. Also, $(A \circ f)\,(x \vee y,q) = A(f(x \vee y),q) = A(f(y) \vee f(x),q) = A(f(x) \vee f(y),q) = A(f(y \vee x),q) = (A \circ f)(y \vee x,q)$. Therefore, $(A \circ f)(x \vee y,q) = (A \circ f)(y \vee x,q)$, for all x and y in R and q in Q. $(A \circ f)(x \wedge y,q) = A(f(x \wedge y),q) = A(f(y) \wedge f(x),q) = A(f(x) \wedge f(y),q) = A(f(y \wedge x),q) = (A \circ f)(y \wedge x,q)$. Therefore, $(A \circ f)(x \wedge y,q) = (A \circ f)(y \wedge x,q)$, for all x and y in R and q in Q. Hence $A \circ f$ is an (Q,L)-fuzzy normal ℓ-subsemiring of a ℓ-semiring R.

5. Conclusion

In the study of the structure of a fuzzy algebraic system, we notice that Q-fuzzy with special properties always play an important role. In this paper, we define (Q,L)-fuzzy normal ℓ -subsemirings of a ℓ -semiring and investigate some important results. We hope that the research along this direction can be continued, and in fact, this work would serve as a foundation for further study of the theory of semiring, it will be important to complete more hypothetical exploration to set up an overall structure for the commonsense application.

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