American Journal of Mathematical and Computer Modelling
2022; 7(4): 49-54
http://www.sciencepublishinggroup.com/j/ajmcm
doi: 10.11648/j.ajmcm.20220704.11
ISSN: 2578-8272 (Print); ISSN: 2578-8280 (Online)

A Comparative Analysis of Ordinary Least Squares and Quantile Regression Estimation Technique

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To cite this article:

Runyi Emmanuel Francis, Maureen Tobe Nwakuya. A Comparative Analysis of Ordinary Least Squares and Quantile Regression Estimation Technique. *American Journal of Mathematical and Computer Modelling*. Vol. 7, No. 4, 2022, pp. 49-54.
doi: 10.11648/j.ajmcm.20220704.11

Received: October 30, 2022; **Accepted**: November 18, 2022; **Published**: November 30, 2022

Abstract: This study explores quantile regression estimation technique and its practicality in regression analysis; hence we provide a comparative study in view of quantile regression as an alternative to the traditional ordinary least squares regression. Although the ordinary least squares (OLS) model examines the relationship between the independent variable and the conditional mean of the dependent variable, whereas the quantile regression model examines the relationship between the independent variable and the conditional quantiles of the dependent variable. Quantile regression overcomes various problems associated with OLS. First, quantile regression is defined and its advantages over ordinary least squares regression are illustrated. Also, specific comparisons are made between ordinary least squares and quantile regression estimation methods. Lastly, both estimation techniques were applied on a real life data and the results obtained from the analysis of two types of datasets in this study suggests that quantile regression provides a richer characterization of the data giving rise to the impact of a covariate on the entire distribution of the response variables as the effect can be very different for different quantiles. Quantile regression therefore gives an efficient and more complete view of the relationship amongst variables, hence, suitable in examining predictors effects at various locations of the outcome distribution.

Keywords: Regression, Ordinary Least Squares Regression, Quantile Regression, Mean Square Error, Variance

1. Introduction

Regression analysis is a cornerstone of statistical analysis which is concerned with describing the dependence and relationship between a response variable and one or more explanatory variables [12] with the view of estimating and or predicting the (population) mean or average value [10].

In Linear regression the regression parameters can be estimated using the ordinary least squares method which estimates regression parameters based on minimizing the sum of squared residuals (residuals are the difference between the observed response and the predicted response). The ordinary least squares regression method yields parameter β_0 and β_1 using $\bar{x}$ to represent the mean value of the predictor variable, $\bar{y}$ to represent the mean value of the response variable [8].

Inferences on ordinary least squares regression requires specific assumptions be made about the distribution of the error which are: normal ($E(\varepsilon_i) = 0$) , homoscedasticity ($V(\varepsilon_i) = \sigma^2$), and uncorrelation ($Cov(\varepsilon_i\varepsilon_j) = 0$). Now, if one or more of the listed assumptions are violated the result and inference from the ordinary least square regression analysis can be impacted and becomes unreliable.

Quantile regression (QR) fits in better when one or more of the listed assumptions are violated in the ordinary least square regression analysis [3]. This positions the question of relationship between the dependent variable and the independent variable at any quantile of the conditional distribution function and gives a complete information about the relationship between the response variable and the covariates on the entire conditional distribution function, and makes no distributional assumption about the error term in the model [6]. When the mean regressions are in fact significantly different, the quantile regression can suggest which part of the conditional distributions differ [7]. Quantile regression is similar to linear regression in that it is used to gain an understanding of how a set of predictor variables are related to a continuous response variable [11]; however, there are several differences in the calculation, assumptions and interpretation when comparing quantile regression to linear regression.

Koenker and Basset [9], introduced quantile regression as a robust alternative to the ordinary least squares as a way to model the quantile of a response variable, the median, conditional on a certain value of a set of predictors. As the quantile is estimated conditional on the set of predictors, it commonly referred to as the conditional quantile [1]. The quantile regression unlike the least squares method has the advantage of providing a detailed picture of the relationship between the covariates and the dependent variable.

Hence, this study aims at bringing to the realization the importance of using quantile regression method in the analysis of data and also establish that quantile regression is an efficient alternative to ordinary least squares.

2. Materials and Methods of Research

Given the nature of most dataset in econometrics, biostatistics, etc., it is hence appropriate to find out which method of analysis is ideal to give a holistic understanding of the effect of covariates on the conditional distribution of the response variable.

This study is interested in comparing the performance of ordinary least squares and quantile regression model in estimation technique. Two categories of datasets will be considered for analysis. The OLS technique and the quantile regression model will be used in the estimation process.

2.1. Linear Regression

Linear regression is used for special class of relationship, those that can be described by straight line [11]. Assuming a given dataset consist of dependent and independent variable and there exist a relationship between the dependent and independent variable then the general linear model can be of the form:

$$y_i|x_i = \beta_0 + \beta_1 x_i + \varepsilon_i \quad (1)$$

where β_0 and β_1 are regression parameters, and indicates the mean response when x_i= 0, and the change in the mean response associated with a one unit increase in the independent variable x_i, respectively [8]. The error term is represented by ε_i and is defined as the difference from the population mean response, associated with the corresponding x_i value, for the individual observation. The matrix form is given as:

$$y_i = x_i\beta + \varepsilon_i \quad (2)$$

where $= (y_1, y_2, \dots, y_n)$; $\varepsilon = (\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n)^T$; $\beta = (\beta_1, \beta_2, \dots, \beta_n)^T$ and $x = \begin{pmatrix} 1 & x_{11}\, x_{12} & \cdots & x_{nn} \\ \vdots & \cdots\cdots & \cdots & \cdots \\ 1 & x_{n1}\, x_{n2} & \cdots & x_{nn} \end{pmatrix}$

OLS minimizes the square distance between the observed and the predicted dependent variable y:

$$s(\beta) = \sum_{i=1}^{N}(y_i - x_i^T\beta)^2 = (y - x\beta)^T(y - x\beta) \quad (3)$$

Differentiating with respect to β setting to zero, we realize that $\hat{\beta}$ satisfies

$$x^T x\beta = x^T y \quad (4)$$

The OLS method of estimating β can be estimated by $\hat{\beta}$ which is given by:

$$\hat{\beta} = (x^T x)^{-1} x^T y \quad (5)$$

The dependent variable can be predicted by $\hat{y} = x_i^T\hat{\beta}$ and the error term by $\hat{\varepsilon}_i = y_i - x_i^T\beta$. $\hat{\varepsilon}_i$ is the residual. Recall that errors are normally distributed with mean of zero constant variance and are uncorrelated. Also, observations are independently and identically distributed (IID) and there is a linear functional relationship between dependent and explanatory variables [2].

2.2. Quantile Regression Method

A regression technique that requires a quantitative variable as the predicted variable and that accepts any type of predictor variable is quantile regression [13]. Quantile Regression presented by Koenker and Bassett [9] provides an alternative to ordinary least squares regression. Estimating conditional quantiles at various points of the distribution of the dependent variable will allow us trace out different marginal response of the dependent variable to change in the explanatory variables at these points [12]. The quantile regression model is given as:

$$Q_{y_i|x_1}(p) = \beta_0^{(q)} + \beta_1^{(q)} x_i + \varepsilon_i^{(q)} \quad (6)$$

Here $\beta_0^{(p)}$ and $\beta_1^{(p)}$ represents the p^{th} quantile of the response variable when x=0 and the change in the p^{th} quantile for a one unit increase in the predictor variable, respectively.

The matrix form is given as:

$$y_i = x_i'\beta_q + \varepsilon_{iq} \quad (7)$$

with $Quant_q(x_i|y_i) = x_i'\beta_q$
where y is the dependent variable, x is a vector of regressors, β is a vector of parameters to be estimated and ε is a vector of residuals. $Quant_q(x_i|y_i)$ denotes the q^{th} regression quantile, $0 < q < 1$ solves the problem below;

$$Q(\beta_q) = min_\beta \sum_{i=1}^{n}\left[|y_i - x_i\beta_q|\right] = min\left[\sum_{i:\, y_i \geq x_i\beta} q|y_i - x_i\beta_q| + \sum_{i:y_i<x_i\beta}(1-q)|y_i - x_i\beta_q|\right] \quad (8)$$

2.3. Goodness of Fit Using Pseudo R^2

The simple quantile model with n independent variables is given as:

$$Q_{y_i|x_1}(q) = \beta_0^{(q)} + \beta_1^{(q)} x_i + \dots + \beta_n^{(q)} x_n \quad (9)$$

The indicator of the goodness of fit for the model can be predicted with the Pseudo R^2 as:

$$Pseudo\ R_p^2 = 1 - \frac{RAWS_q}{TASW_q} \quad (10)$$

where:

$$RAWS_q = \sum_{y_i \geq \hat{\beta}_0(q)+\hat{\beta}_1(q)x_i+\cdots+\hat{\beta}_n(q)x_n} q\left|y_i - \hat{\beta}_0(q) - \hat{\beta}_1(q)x_i - \cdots - \hat{\beta}_n(q)x_n\right|$$

$$+\sum_{y_i < \hat{\beta}_0(q)+\hat{\beta}_1(q)x_i+\cdots+\hat{\beta}_n(q)x_n} (1-q)\left|y_i - \hat{\beta}_0(q) - \hat{\beta}_1(q)x_i - \cdots - \hat{\beta}_n(q)x_n\right| \quad (11)$$

and

$$TASW_q = \sum_{y_i \geq q} q|y_i - \hat{q}| + \sum_{y_i < q}(1-q)|y_i - \hat{q}| \quad (12)$$

The value of $RAWS_q$ (Residual Absolute Sum of Weighted) is always less than the value of the $TASW_q$ (Total Absolute Sum of Weighted) so that the Pseudo R_q^2 will range from 0 to 1. The closer the value of the Pseudo R^2 is to one, the better the model will be. However, the qualities of Pseudo R^2 can not be used to test the overall goodness of fit for the model, it can be used to test the merits of the selected quantile [4].

2.4. Mean Square Error (MSE)

The parameter estimate obtained is said to be good if it has a small bias and small variance. Therefore, to see the goodness of estimating the parameters based on the bias and variance values simultaneously, represented in the value of the Mean Square Error (MSE) [5] given as:

$$MSE\left(\hat{\beta}_i(q_p)\right) = Var\left(\hat{\beta}_i(q_p)\right) + Bias\left(\hat{\beta}_i(q_p)\right)^2 \quad (13)$$

where:

$MSE\left(\hat{\beta}_i(q_p)\right)$: value of MSE for $i = 1,2, \dots j\ and\ q_p = quantile\ q = 1,2, \dots, k$

$Var\left(\hat{\beta}_i(q_p)\right)$: variance of selected quantile $Var\left(\hat{\beta}_i(q_p)\right) = \frac{n\sum_{j=1}^{n}(\hat{\beta}_i(q_p))^2 - (\sum_{j=1}^{n}(\hat{\beta}_i(q_p)))^2}{n(n-1)}$

$Bias\left(\hat{\beta}_i(q_p)\right)$: the value of the bias for selected quantile is obtained from the mean of the difference of the expected value and the estimated value or: $Bias\left(\hat{\beta}_i(q_p)\right) = \frac{1}{n}\sum_{j=1}^{n}\left(\hat{\beta}_i(q_p)\right) - \left(\beta_i(q_p)\right)$

3. Application, Analysis, Results & Discussion

We present the analysis and discussion of the results obtained from two sets of data in order to validate the analytical power of Quantile Regression. Two linear models were used for analysis. R Studio software was used to analyze both datasets.

Illustrative Data 1: Relationship between high blood pressure and body mass index of patients in the University of Calabar Teaching Hospital (UCTH), Cross River state.

Dependent Variable (Y): High Blood Pressure.

Independent Variable (X): Body Mass Index.

From table 1, the Jarque Bera P-value, Y which is the dependent variable does not follow a normal distribution. To this regard, the information above provides the preliminary justification for engaging quantile regression.

Table 1. *Descriptive Statistics.*

	BMI	BP
Mean	1.313039	127.0196
Median	1.285000	124.0000
Maximum	2.390000	208.0000
Minimum	0.810000	94.00000
Std. Dev.	0.257839	18.18441
Skewness	1.260221	1.406322
Kurtosis	6.085016	6.740396
Jarque-Bera	67.44732	93.08147
Probability	0.000000	0.000000

Summary Statistics.

Table 2. *Estimation of OLS Parameters.*

	Estimate	Std Error	t-value	P (>|t|)
Intercept	0.7663	0.3026	2.532	0.01293
X	0.0049	0.0012	4.031	0.00011***
S. E = 0.2211; R-Sq = 28%; R-Sq (Adjusted) = 26.5%; AIC= 85.5591				

Ordinary Least Square Estimate are shown.

Table 2 shows that the percentage of determination is low as a result of the relationship between the dependent variable (blood pressure) and the independent variable (body mass index). The independent variable explained 28% of the variance of the dependent variable while the remaining 72% is explained by external (other) factors. Also, the coefficient $\beta_0 = 0.766$ while $\beta_1 = 0.0049$ is significant with 0.00011 probability value.

The ordinary least square regression equation is

$$Blood\ Pressure = 0.766 + 0.0049\ BMI$$

Table 3. *Regression Analysis.*

	Linear Regression	Quantile Regression		
	Mean	**0.25 Quantile**	**0.50 Quantile**	**0.75 Quantile**
Intercept	0.7663	0.8119	0.842	0.6700
X	0.0049	0.0026	0.0044	0.0061
Pseudo R^2		0.7657	0.5291	0.7391
AIC		78.4395	84.9314	76.0153

Model Estimate are shown.

At 25th quantile $intercept = 0.8119$ which is the predicted value of the 25th quantile of Body Mass Index. $\beta_{(0.25\ quantile)} = 0.0026$ indicates the rate of change of the 25th quantile of the response variable distribution per unit changes in the value of the regressors (BMI). In other words, at the 25th quantile of the dependent variable, a unit increase in the BMI increases the blood pressure of the patient by 0.0026.

At 50th quantile $intercept = 0.8420$ which is the predicted value of the 50th quantile of Body Mass Index. $\beta_{(0.50\ quantile)} = 0.0035$ indicates the rate of change of the 50th quantile of the response variable distribution per unit change in the value of the regressors (BMI). In other words, at the 50th quantile of the dependent variable, a unit increase in the BMI increases the blood pressure of the patient by 0.0035.

At 75th quantile $intercept = 0.6700$ which is the predicted value of the 75th quantile of Body Mass Index. $\beta_{(0.75\ quantile)} = 0.0061$ indicates the rate of change of the 75th quantile of the response variable distribution per unit change in the value of the regressors (BMI). In other words, at the 75th quantile of the dependent variable, a unit increase in the BMI increases the blood pressure of the patient by 0.0061.

Table 4. *Descriptive Statistics.*

	CO_2	GDP	POP	TR
Mean	0.317827	1.29E+10	2.595975	54.89554
Median	0.282309	5.25E+09	2.554408	48.63636
Maximum	0.680000	6.56E+10	3.213951	116.0484
Minimum	0.181470	1.22E+09	1.856640	6.320343
Std. Dev.	0.115701	1.78E+10	0.316475	26.33009
Skewness	1.540976	1.822173	-0.072891	0.295115
Kurtosis	5.094683	4.954515	2.493227	2.417675
Jarque-Bera	34.13673	42.04091	0.683592	1.690038
Probability	0.000000	0.000000	0.710493	0.429549

For the independent variable, ordinary least squares suggest that the variable has a positive influence on the dependent variable (High Blood Pressure) with an estimate of $\beta_1 = 0.0049$. This implies that high blood pressure increases as the level of obesity increases. Quantile regression as well confirms this statement but gives a more expanded understanding of the influence of the independent variable. The influence at the 0.75 quantile with estimates $\beta_{(0.75)} = 0.0061$ seems to be stronger than is suggested by the ordinary least squares.

The goodness of fit for each quantile regression model is represented by the value of Pseudo R^2, as showed in table 3. All Pseudo R^2 values obtained here are more than 70% except for the 50th quantile = 0.5291 therefore indicating that the proposed model at the 25th and 75th quantile is more adequate and could be accepted. The R^2 value = 0.7657, that is 76.57% of the dependent variable can be explained by the independent variable with an AIC = 78.4395 at the 25th quantile. The R^2 value = 0.5291, that is 52.91% of the dependent variable can be explained by the independent variable with an AIC = 84.9314 at the 50th quantile and the R^2 value = 0.7391, that is 73.91% of the dependent variable can be explained by the independent variable with an AIC = 76.0153 at the 75th quantile. Therefore, the quantiles (25th and 75th) with smaller AIC values presents the more adequate model that could be accepted for forecasting or prediction as compared to the AIC value of the OLS = 85.5591.

Also, we determined the value of the MSE (mean Square Error) to ensure that parameter estimated have small bias and small variance. Table 5 below presents the MSE values of the quantile regression method for the parameters estimated at corresponding points.

Table 5. *The MSE, Bias and Variance Value for OLS and different quantiles.*

	MSE	Bias	Variance
OLS	0.6052	0.2992	0.5087
25th Quantile	0.4130	0.2507	0.3883
50th Quantile	0.5329	0.3025	0.4867
75th Quantile	0.4577	0.2299	0.4523

Table 5 above gives information that the parameter estimates have small MSE at the 25th and 75th quantiles as compared to the 50th quantile and the OLS, so also the bias and variance value at the 25th and 75th quantiles.

Illustrative Date 2: Factors influencing carbon emission at different levels of emission.

Dependent Variable (Y): Carbon Emission.

Independent Variable (X1): GDP.

Independent Variable (X2): Trade Openness.

Independent Variable (X3): Population.

From table 3, the Jarque Bera P-value, Y which is the dependent variable does not follow a normal distribution. To this regard, the information above provides the preliminary justification for engaging quantile regression.

Table 6. *Estimation of OLS Parameters.*

Variable	Estimate	Std Error	t-value	P (>|t|)
Intercept	-6.1369	0.4645	-13.2127	0.0000
X1	0.2015	0.01880	11.7794	0.0000
X2	-0.3560	0.45418	-2.3088	0.0247
X3	0.0711	0.03253	2.1859	0.331
S= 0.1344; R-Sq = 83.05%; R-Sq (Adjusted) = 82.12%; AIC = 70.3499				

Ordinary Least Square Estimate are shown.

Table 6 shows that the percentage of determination is high as a result of the relationship between the dependent variable (Y) and the independent variables (X1, X2, X3). The independent variables explained 83.05% of the variance of the dependent variable while the remaining 16.98% is explained by external (other) factors. Also, the coefficient $\beta_0 = -6.1369$ while $\beta_1 = 0.2215$ is significant with 0.0000 probability value and $\beta_2 = -0.3560$, $\beta_3 = 0.0711$ are not significant.

The ordinary least square regression equation is

$$Carbon\ Emission = -6.1369 + 0.2215\ GDP - 0.3560\ TO + 0.0711\ POP$$

Table 7. *Regression Analysis.*

	Linear Regression		**Quantile Regression**					
	Mean	**P (>|t|)**	**0.25 Quantile**	**P (>|t|)**	**0.50 Quantile**	**P (>|t|)**	**0.75 Quantile**	**P (>|t|)**
Intercept	-6.1369	0.0000	-5.9343	0.0000	-6.1605	0.0000	-5.8112	0.0000
X1	0.2015	0.0000	0.1965	0.0000	0.2196	0.0000	0.2109	0.0000
X2	-0.3560	0.0247	-0.3859	0.0096	-0.3636	0.0923	-0.2951	0.2343
X3	0.0711	0.331	0.1406	0.0672	0.1055	0.1454	0.0523	0.1208
Pseudo R^2			0.7133		0.4981		0.7033	
AIC			66.1729		68.6137		65.0418	

Model Estimate are shown.

At 25th quantile, a percentage change in X1 (GDP) increases Y (Carbon Emission) by 0.196%. Also, in interpreting the effect of X1 (GDP) on Y (Carbon Emission) at the 50th quantile, at the median, a percentage change in X1 (GDP) increases Y (Carbon Emission) by 0.22%. The effect of X1 (GDP) on Y (Carbon Emission) at 0.75 quantile shows that a percentage change in X1 (GDP) increases Y (Carbon Emission) by 0.21%.

At 25th quantile $intercept = -5.9343$ which is the predicted value of the 0.25 quantile of GDP. $\beta_{(0.25\ quantile)} = 0.1965$ indicates the rate of change of the 25th quantile of the response variable distribution per unit change in the value of the regressors (GDP). In other words, at the 25th quantile of the dependent variable, a unit increase in the GDP increases the Carbon Emission by 0.1965.

At 50th quantile $intercept = -6.1605$ which is the predicted value of the 50th quantile of GDP. $\beta_{(0.50\ quantile)} = 0.2196$ indicates the rate of change of the 50th quantile of the response variable distribution per unit change in the value of the regressors (GDP). In other words, at the 50th quantile of the dependent variable, a unit increase in GDP increases the Carbon Emission by 0.2196.

At 75th quantile $intercept = -5.8112$ which is the predicted value of the 75th quantile of GDP. $\beta_{(0.75\ quantile)} = 0.2109$ indicates the rate of change of the 75th quantile of the response variable distribution per unit changes in the value of the regressors (GDP). In other words, at the 75th quantile of the dependent variable, a unit increase in GDP increases the Carbon Emission by 0.2109.

For the independent variable, ordinary least squares suggest that the variable GDP has a positive influence on the dependent variable (Carbon Emission). This implies that Carbon Emission increases as the GDP level increases. Quantile regression as well confirms this statement but gives a more expanded understanding of the influence of the independent variable. The influence at the 0.75 quantile with estimates $\beta_{(0.75)} = 0.2109$ seems to be stronger than is suggested by the ordinary least squares.

The goodness of fit for each quantile regression model is represented by the value of Pseudo R^2, as showed in table 7. All Pseudo R^2 values obtained here are more than 70% except for the 50th quantile = 0.498122 therefore indicating that the proposed model at the 25th and 75th quantile is adequate and could be accepted. The R^2 value = 0.7133, that is 71.33% of the dependent variable can be explained by the independent variable with an AIC = 66.1729 at the 25th quantile. The R^2 value = 0.4981, that is 49.81% of the dependent variable can be explained by the independent variable with an AIC = 68.6137 at the 50th quantile and the R^2 value = 0.7033, that is 70.33% of the dependent variable can be explained by the independent variable with an AIC = 65.0418 at the 75th quantile. Therefore, the quantiles (25th and 75th) with smaller AIC values presents the more adequate model that could be accepted for forecasting or prediction as compared to the AIC value of the OLS = 70.3499.

Also, we determined the value of the MSE (mean Square Error) to ensure that parameter estimated have small bias and small variance. Table 8 below presents the MSE values of the quantile regression method for the parameters estimated at corresponding points.

Table 8. *The MSE, Bias and Variance Value for different quantiles.*

	MSE	**Bias**	**Variance**
OLS	0.5322	0.6996	0.0617
25th Quantile	0.4130	0.4024	0.0288
50th Quantile	0.5329	0.7441	0.0792
75th Quantile	0.4577	0.5582	0.0397

Table 8 above gives information that the parameter estimates have small MSE at the 25th and 75th quantiles as compared to the 50th quantile and the OLS, so also the bias and variance value at the 25th and 75th quantiles.

4. Conclusion

Frequently, the research question can define whether quantile regression is desired over linear regression, as most areas of research is interested with explicit quantiles of the conditional distribution of an outcome variable rather than simply the conditional mean. Linear regression is mostly used and easily understood method of analysis and especially when the assumptions are met to aid give full description of the relationship between dependent and independent. Quantile regression offers a widespread approach for completing the regression picture as it goes beyond the primary goal of determining just the conditional mean, and enables one to spot reasonable questions of the relationship between the response variable and explanatory variable at any given quantile of the conditional distribution.

This study used quantile regression and the traditional ordinary least square regression method for the analysis of two sets of datasets. The estimates across the quantile regression allows us to study the impact of predictors on different quantiles of the dependent variable, and hence provide us with a complete picture of the relationship between the dependent and independent variables. This study also presents results that shows that quantile regression is able to produce small value of MSE, bias, variance and AIC hence it could be concluded that quantile regression methods produce efficient results and could be an alternative technique against the traditional ordinary least square regression in investigating the relationship between variables.

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